08:35:12 OCA PAD INITIATION - PROJECT HEADER INFORMATION 12/02/92

Active

Project #: E-24-X01 Cost share #: Rev #: 0
Center # : 10/24-6-R7692-0A0 Center shr #: OCA file #:
Work type : RES
Contract#: AGREEMENT SIGNED 11/18/92 Mod #: Document : AGR
Prime #: Contract entity: GTRC

Subprojects ? : N CFDA: N/A
Main project #: PE #: N/A

Project unit: ISYE Unit code: 02.010.124
Project director(s):
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BARNHART C ISYE (404)-
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Sponsor/division names: AMERICAN AIRLINES DECIS TECHN / DALLAS, TX
Sponsor/division codes: 200 / 110

Award period: 921201 to 931130 (performance) 931130 (reports)

Sponsor amount	New this change	Total to date
Contract value	25,000.00	25,000.00
Funded	25,000.00	25,000.00
Cost sharing amount		0.00

Does subcontracting plan apply ?: N

Title: COLUMN GENERATION FOR AIRLINE PROBLEMS

PROJECT ADMINISTRATION DATA

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Security class (U,C,S,TS) : U ONR resident rep. is ACO (Y/N): N
Defense priority rating : N/A N/A supplemental sheet
Equipment title vests with: Sponsor X GIT
NONE PROPOSED.
Administrative comments -
INITIATION OF ONE-YEAR "INDUSTRIAL-MATCHING-FUNDS" GRANT UNDER NSF GRANT NO.
E-24-666.

GEORGIA INSTITUTE OF TECHNOLOGY
OFFICE OF CONTRACT ADMINISTRATION

NOTICE OF PROJECT CLOSEOUT

Closeout Notice Date 02/15/94

Project No. E-24-X01 Center No. 10/24-6-R7692-0A0

Project Director NEMHAUSER G L School/Lab ISYE

Sponsor AMERICAN AIRLINES DECIS TECHN/DALLAS, TX

Contract/Grant No. AGREEMENT SIGNED 11/18/92 Contract Entity GTRC

Prime Contract No. ______

Title COLUMN GENERATION FOR AIRLINE PROBLEMS

Effective Completion Date 931130 (Performance) 931130 (Reports)

Closeout Actions Required:	Y/N	Date Submitted
Final Invoice or Copy of Final Invoice	Y	
Final Report of Inventions and/or Subcontracts	N	
Government Property Inventory & Related Certificate	N	
Classified Material Certificate	N	
Release and Assignment	N	
Other	N	

Comments ______

Subproject Under Main Project No. ______

Continues Project No. ______

Distribution Required:

Project Director	Y
Administrative Network Representative	Y
GTRI Accounting/Grants and Contracts	Y
Procurement/Supply Services	Y
Research Property Managment	Y
Research Security Services	N
Reports Coordinator (OCA)	Y
GTRC	Y
Project File	Y
Other	N
	N

E-24-X01
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Report to American Airlines on Crew Pairing Project

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September 17, 1993

1 Summary of Results

Most solution procedures for the airline crew scheduling problem use a set partitioning IP formulation with a row for each segment in the schedule and columns representing the pairings. The constraints in the formulation enforce that each flight is contained in exactly one pairing in a feasible solution. Tremendous progress has been made on solving the LP relaxation of this formulation over a subset of the pairings for large problems (800 flights, 5.5 million pairings), but work still remains on solving the LP over all possible columns and obtaining optimal or provably good integer solutions. Our research primarily addresses the issue of obtaining good integer solutions. We also implicitly consider the full-sized LP.

Our work focuses on using a different formulation of the problem to obtain a stronger LP bound. Strong LP relaxations improve the efficiency of branch-and-bound schemes and improve the chances that the optimal solution to the LP relaxation will be integral or nearly integral. Our results confirm this observation. In particular, we are able to solve an integer program by solving a linear program without performing branch-and-bound. Moreover, by terminating the LP early we get an integer solution and a tolerance guaranteeing its quality. In our experiments this solution was optimal. These results are

problem	flights	duties	pairings
1	144	1795	93,851
2	174	2716	467,671
3	202	3203	1,878,614
4	253	4865	5,833,004

Table 1: Problem Characteristics

very encouraging, but further research is required to improve running time and to solve larger problems.

The basic idea of our approach is to decompose the pairing problem into two problems. First, we decide on optimal duty periods to cover the segments. Then we decide on optimal pairings, to cover the duties. Since these duties may not provide good pairings, duty prices are returned to the segment/duty problem to yield new duties. This process continues until optimal pairings are found.

Computational trials were performed using flight schedule data for a single aircraft fleet provided by AADT. All possible duty periods were enumerated over the flights in each test problem. The sizes of the test problems are shown in Table 1. Table 1 illustrates the dramatic growth in problem size as the number of flights increases. Although computational trials have only been performed on the first two problems, the last two problems are listed to illustrate the growth in the number of pairings which is faster than we previously believed.

The results of computational trials on the sample problems are shown in Table 2. Formulation is the type of model used with "new" being our proposed model and "standard" being the traditional set partitioning model. Columns is the total number of columns present in the final master problem matrix. Only a small fraction of the total number of columns needed to be explicitly considered. Gap is the difference between the optimal value of the LP and a lower bound provided by the duty period set subproblem. Because of the tailing effect typical of column generation solution procedures, we found it advantageous to cease column generation once the gap was sufficiently small. The set of trials for Problem 1 illustrate the amount of computational effort necessary to obtain a guarantee that the current solution is within the

problem	formulation	columns	gap	total time
1	new	3884	1.0%	1671
1	new	4619	0.5%	2898
1	new	4893	0.25%	3037
1	new	5696	0.1%	3618
1	new	9287	0.0%	10666
2	new	13279	0.2%	20677
1	standard	93851	0.0%	2994

Table 2: Computational Results

tolerance specified by gap of the optimal solution. Time is the total CPU time in seconds on an IBM RS6000/550 necessary to solve the problem using MINTO Version 1.4d and CPLEX Version 2.1. The last result reported for Problem 1 is the total time necessary to prove integer optimality using the branch and bound on the traditional set partitioning formulation with all the pairings enumerated at the beginning. Problem 2 had too many pairings to be solved directly by the traditional approach.

For both problems 1 and 2, the solution to the LP relaxation of the new formulation was integral. The LP relaxation of the traditional set partitioning formulation did not provide an integer solution for Problem 1. Nearly 75 % of the variables that take on positive values in the optimal solution to the traditional LP relaxation were fractional. It was necessary to branch 11 times before the first integer solution was found.

We wish to stress the fact that these computational results are of a preliminary nature. The solution times for the new formulation are not as competitive as we would like, nor have we been able to solve problems as large as we would like. However, if we can speed up the convergence of the LP, we expect to produce LP solutions where more of the variables take on integer values than they would in the optimal solution to the LP relaxation of the traditional formulation. Given these less fractional solutions, integrality should be much easier to obtain through branching than is the case for the traditional formulation.

Regarding convergence of the LP, the duty period set subproblem seems to be the bottleneck. We have an efficient constrained shortest path algorithm that identifies all of the pairings with favorable reduced costs at one pass, but the current duty period set subproblem only produces a single duty period set at each call. This is not practical for large problems. Further work needs to be done on finding many of these duty sets at one time. The tradeoff between the two subproblems is also not well understood. In the current implementation, pairings are generated after every fifth call to the duty period set subproblem. More trials need to be run to evaluate the interaction between the frequency of calls to each subproblem type and the speed of convergence of the master LP. In addition, now that we have the shortest path pairing generator in hand, we would like to use the constrained shortest path procedure to generate columns dynamically for the traditional formulation. Through these trials, we can evaluate the two formulations on a more consistent basis.

2 Outline of New Formulation and Algorithm

To give some insight into the structure of this new formulation, we give a brief history of its evolution. We began with a formulation that had a row for each possible duty period and two types of columns. One set of columns represents feasible pairings and the other represents sets of duty periods that partition the flights in the schedule. The constraints enforce that exactly one set of duties is chosen and every duty in the chosen set is covered by a pairing in the solution. We attempted to solve this formulation using a column generation approach. Duty period set columns were generated by solving a minimum reduced cost set partitioning problem to find a set of duties that partition the flights. Pairing columns were generated using a constrained shortest path procedure.

The difficulty with this first formulation was slow convergence of the LP relaxation. Part of the reason for this slow convergence is that there are a large number of feasible duty period sets for a given flight schedule. However, many of these sets cannot be partitioned into sets of legal pairings. Thus, the solution procedure spends a large fraction of the execution time generating columns that do not improve the LP solution.

To speed up the convergence of the column generation scheme for this formulation, we composed an alternate formulation motivated by work done on multicommodity flow problems. We can think of the idea as follows: rather than having decision variables that represent duty period sets in the master problem, we can choose one duty period set that partitions the flights and call it our *key set*; we then add decision variables that represent possible modifications of the key set to form other duty period sets. We call these modifications *duty exchanges* since they can be thought of as exchanging a subset of the duties in the key set for a new set of duty periods. We refer to the members of the key set as *key duties* and duty periods not in the key set as *new duties*. The key duties and the new duties in a given exchange must partition the same subset of the flight legs.

This modified formulation contains columns representing each possible exchange as well as columns representing the pairings. There is a constraint allowing at most one exchange to be chosen. Also, there are constraints enforcing that each key duty period must either be exchanged for some other duty (or duties) that cover the same flights, or a pairing must be chosen that uses the key duty period. For the new duty periods, there are constraints that force a pairing to be chosen that covers the new duty if the duty is used in an exchange that is chosen in the solution. Duty exchanges can be generated using the same subproblem used to generate duty period sets.

Although we might obtain somewhat faster convergence after making this modification, since the two formulations have the same number of columns, it is unlikely that we will obtain enough improvement from the key set modification alone to solve large problem instances. For this reason, we relax the formulation. To motivate the relaxation, note that some exchanges can be expressed as the combination of two or more other exchanges. We refer to these exchanges as *compound* exchanges. Exchanges that cannot be represented as the combination of other exchanges are referred to as *simple* exchanges. By omitting the constraint allowing at most one exchange to be chosen and by including only simple exchanges in the formulation we can decrease dramatically the number of candidate exchange columns without eliminating any feasible solutions to the problem. However, we may introduce new fractional feasible solutions. In general, the value of the LP bound obtained from the relaxed key set formulation will be somewhere between the LP bound obtained from the traditional set partitioning formulation and the bound from the first modified formulation.